Interest Rate Sensitivities of Bond Risk Measures

Timothy Falcon Crack and Sanjay K. Nawalkha

Author Digest

Building on previous research into the sensitivities of bond risk measures, we present a simple expression for the sensitivity of duration, convexity, and higher-order bond risk measures to nonparallel changes in the shape of the yield curve. Although researchers have analyzed the sensitivity of a bond's duration to changes in the bond's yield, little is known about the interest rate sensitivity of duration, convexity, and so on to changes in level, slope, and curvature of the term structure. The subject is important because up to 95 percent of returns to portfolios of U.S. Treasury securities are explained by term-structure level, slope, and curvature shifts—and these shifts can be quite extreme in volatile interest rate environments.

We captured these parameters of term-structure shape by using a simple polynomial representation of the continuously compounded spot yield curve. Given a noninfinitesimal, nonparallel shift in the yield curve, we were able to derive closed-form expressions for the resulting changes in bond risk measures as a function of changes in the level, slope, and curvature of term structure and as a function of the bond risk measures themselves.

Our framework enabled us to answer questions that are relevant to the work of managers who are required to maintain target durations for their bond portfolios and who wish to know how sensitive their bond risk positions are to general interest rate changes: How does the duration of a bond change with

---

Timothy Falcon Crack is assistant professor of finance at Indiana University. Sanjay K. Nawalkha is associate professor at the University of Massachusetts at Amherst.

respect to a change in the slope of the term structure? How does the convexity of a bond change with respect to a change in the level of the term structure? Do the duration and convexity of a barbell portfolio change more rapidly than those of a bullet portfolio? These questions are relevant to managers of fixed-income portfolios and managers of financial institutions.

Shifts in term-structure level, slope, and curvature are not independent. For example, increases in level tend to be associated with decreases in slope. We used such interrelationships to derive a simple but realistic numerical example of the effect of a noninfinitesimal, nonparallel term structure shift on a bullet bond and two barbell bonds. We found that if we ignore the slope and curvature shifts and account only for the level shift, we seriously misestimated the effect of the full term-structure shift on bond duration measures for the barbell bonds. The percentage error we made became larger as the cash flow spacing of the barbell became wider. When we added two terms (i.e., slope to level) and then three terms (i.e., curvature to level and slope), the magnitude of our estimation errors decreased substantially. Therefore, accounting for the impact of level shifts alone (i.e., parallel shifts) is not sufficient when estimating the effect of changes in term-structure shape on bond risk measures.

We also note one simple result: Although the bonds in our numerical example all had the same initial price and duration, the effect of the nonparallel shift in term structure on their prices was quite different. This outcome is a simple reminder that practitioners must look beyond parallel term-structure shifts when analyzing bonds.